International Journal of Theoretical Physics, Vol. 46, No. 6, June 2007 (© 2007)
DOI: 10.1007/s10773-006-9293-y

Massive Charged Quasinormal Modes of a Reissner-Nordström Black Hole

J. F. Chang[1,2,3,5] and Y. G. Shen[1,3,4]

Received August 30, 2006; accepted October 2, 2006
Published Online: January 18, 2007

Using the WKB method and HYBRD program, we evaluate the low-lying massive charged scalar and Dirac field quasinormal modes (QNMs) of a Reissner-Nordström black hole. We discuss the real and imaginary parts of QNMs vary with the charge of black hole, the masses and charges of scalar and Dirac fields.

KEY WORDS: black hole; quasinormal modes; WKB method; low-lying modes.

PACS numbers: 04.70.-s, 04.50.+h, 11.15.-q, 11.25.Hf

1. INTRODUCTION

The perturbations of black holes have been studied for many years. It is well-known that there are three stages during the evolution of the field perturbation in the black hole background: the initial outburst from the source of perturbation, the quasinormal oscillations and the asymptotic tails. The frequencies and damping time of the quasinormal oscillations called "quasinormal mode" (QNM) are determined only by the black hole's parameters and independent of the initial perturbations. QNM frequencies (QNMs) is one of the important and exciting themes in the black hole physics. A great deal of efforts have been devoted to the black hole's QNMs for the possibility of direct identification of black hole existence

[1] Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, People's Republic of China.
[2] Graduate School of Chinese Academy of Sciences, Beijing 100039, People's Republic of China.
[3] National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, People's Republic of China.
[4] Institute of Theoretical physics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China.
[5] To whom correspondence should be addressed at Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, People's Republic of China; e-mail: changjiaf@yahoo.com.cn, ygshen@shao.ac.cn

through gravitational wave detectors in the near future (Nollert, 1999; Kokkotas and Schmidt, 1999).

Many researchers have developed various method to calculate QNMs (see the review articles (Nollert, 1999; Kokkotas and Schmidt, 1999) and references therein). Most of the studies on QNMs discuss only on massless fields (Su and Shen, 2004, 2005a,b; Konoplya and Abdalla, 2005; Zhidenko, 2004, 2006; Cardoso and Lemos, 2003; Cardoso *et al.*, 2004; Daghigh and Kunstatter, 2005; Chang and Shen, 2005; Berti and Kokkotas, 2005; Jing and Pan, 2005a; Jing, 2005; Jing and Pan, 2005b). There are few studies massive QNMs (Simone and Will, 1992; Konoplya and Zhidenko, 2005; Ohashi and Sakagami, 2004; Xue *et al.*, 2002; Cho, 2003; Konoplya, 2006). Wu and Zhao (2004) investigated massless charged Dirac QNMs and Konoplya (2002) studied massive charged scalar ones of a Reissner-Nordström (RN) black hole.

In this paper, we evaluate the QNMs of a RN black hole for massive charged scalar and Dirac fields perturbation. This article consists of the following contents. The massive charged equations in the RN black hole can be reduced into a set of Schrödinger-like equations with a particular effective potential. In Section 2, we compare the different properties of particular potentials between scalar and Dirac fields. Then we calculate the low-lying QNMs using third order WKB method and present the numerical result and discussion in Section 3. A summary is presented in Section 4. Throughout this paper, we use units in which $G = c = M = 1$.

2. PROPERTIES OF THE EFFECTIVE POTENTIAL

The metric of the RN black hole is

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + r^2(d\theta^2 + d\phi^2), \tag{1}$$

here $f(r) = 1 - 2M/r + Q^2/r^2$, Q and M are the electric charge and the mass of the black hole. Let

$$r_\pm = M \pm \sqrt{M^2 - Q^2} \tag{2}$$

be the roots of $f(r) = 0$, and the surface of $r = r_+$ is an event horizon of the the RN black hole. In curved spacetime, the equations of massive charged Dirac equations are

$$\begin{aligned} &\sqrt{2}\left(\nabla_{A\dot{B}} + ieA_{A\dot{B}}\right) P^A + im_p \bar{Q}_{B'} = 0, \\ &\sqrt{2}\left(\nabla_{A\dot{B}} - ieA_{A\dot{B}}\right) Q^A + im_p \bar{P}_{B'} = 0, \end{aligned} \tag{3}$$

where P^A and Q^A are two two-component spinors, the operator $\nabla_{A\dot{B}}$ denotes the spinor covariant differentiation. $\nabla_{A\dot{B}} = \sigma^\mu_{A\dot{B}}\nabla_\mu$, and $\sigma^u_{A\dot{B}}$ are 2×2 Hermitian matrices which satisfy $g_{\mu\nu}\sigma^\mu_{A\dot{B}}\sigma^\nu_{C\dot{D}} = \epsilon_{AC}\epsilon_{\dot{B}\dot{D}}$, where ϵ_{AC} and $\epsilon_{\dot{B}\dot{D}}$ are antisymmetric

Levi-Civita symbols, the operator ∇_μ is covariant differentiation. $A_{A\dot{B}}$ is the electromagnetic field potential spinor components which $A_\mu = (-Q/r, 0, 0, 0)$. m_p is the mass of Dirac fields expressed in Compton wave length. e is the electric charge of Dirac fields. Dirac equations (3) in RN metric reduce to radial and angular parts (Chang and Shen, 2005; Wu and Zhao, 2004; Page, 1976)

$$\Delta^{1/2}\mathcal{D}_0 R_{-1/2} = (\lambda + im_p r)\Delta^{1/2} R_{+1/2},$$
$$\Delta^{1/2}\mathcal{D}_0^{+}(\Delta^{1/2} R_{+1/2}) = (\lambda - im_p r)\Delta^{1/2} R_{-1/2} \tag{4a}$$
$$\mathcal{L}_{1/2} S_{+1/2}(\theta) = -\lambda S_{-1/2}(\theta),$$
$$\mathcal{L}_{1/2}^{+} S_{-1/2}(\theta) = +\lambda S_{+1/2}(\theta), \tag{4b}$$

where $\Delta = f(r)r^2 = r^2 - 2Mr + Q^2$, and operators such as $\mathcal{D}_n$ are defined in Wagh and Dadhic (1985). $\lambda = \pm(j + 1/2)$ is the separation constant determined by the angular equations (4b) (Goldberg *et al.*, 1967), and $j = (2l - 1)/2$ with l positive integer.

By introducing the tortoise coordinate transformation from the radial variable r to the tortoise coordinate r_* which is given by

$$\mathrm{d}r_* = \frac{r^2}{\Delta}\mathrm{d}r, \tag{5}$$

and then the coordinate transformation

$$\mathrm{d}\hat{r}_* = \left(1 + \frac{\Delta}{r^2}\frac{\lambda m_p}{2\omega}\frac{1}{\lambda^2 + m_p^2 r^2} - \frac{eQ}{r\omega}\right)\mathrm{d}r_*, \tag{6}$$

we combine the radial equations (4a) and obtain the pair of one-dimensional Schrödinger-like equations

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}\hat{r}_*^2} + \omega^2\right) Z_\pm = V_\pm Z_\pm, \tag{7}$$

where

$$V_\pm = W^2 \pm \frac{\mathrm{d}W}{\mathrm{d}\hat{r}_*}, \tag{8}$$

$$W = \frac{\Delta^{1/2}\left(\lambda^2 + m_p^2 r^2\right)^{3/2}}{r^2\left(\lambda^2 + m_p^2 r^2\right)(1 - eQ/r\omega) + \lambda m_p \Delta/2\omega}. \tag{9}$$

V_+ and V_- are super-symmetric partners derived from the same potential W and have the same spectra of QNMs (Anderson and Price, 1991). We conerntrate only on V_+ in evaluating following calculation and drop the subscript of V_+.

Corresponding to equation (3), massive charged Klein-Gordon equation in curved spacetime is

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^{\nu}}-ieA_{\nu}\right)\right]\Psi$$
$$-ieA_{\mu}\left[g^{\mu\nu}\left(\frac{\partial}{\partial x^{\nu}}-ieA_{\nu}\right)\right]\Psi=m_p^2\Psi. \tag{10}$$

Setting $\Psi = e^{-i(\omega t - m\phi)}S(\theta)R(r)/r$ and using the tortoise coordinate transformation defined in equation (5), Konoplya (2002) derived the radial and angular equations

$$-\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}S(\theta)}{\mathrm{d}\theta}\right)+\frac{m^2}{\sin^2\theta}S(\theta)=\lambda S(\theta), \tag{11}$$

$$\frac{\mathrm{d}^2R(r)}{\mathrm{d}r_*^2}+[\omega^2-V]R(r)=0. \tag{12}$$

Equation (11) is a spheric harmonics function and the separation constant $\lambda = l(l+1)$, where $l = 0, 1, 2, 3\ldots$ is the angular momentum quantum number. The effective potential in the one-dimensional Schrödinger-like radial equation (12) is

$$V=f(r)\left(\frac{l(l+1)}{r^2}+\frac{2M}{r^3}-\frac{2Q^2}{r^4}+m_p^2\right)+\frac{2\omega eQ}{r}-\frac{e^2Q^2}{r^2}. \tag{13}$$

The sign of the product eQ affects the equations (8) and (13). Taken Q to be always positive, a negative e means e and Q have contrary signs, and vice versa.

Wu and Zhao (2004) have studied the effective potential for charged massless Dirac fields in RN spacetimes in detail. In general speaking, $eQ > 0$ increases the effective potential. In Figs. 1 and 2, we show the dependence of effective potential for scalar and Dirac fields on ω. As ω increases, the effective potential for scalar fields increases slowly for a positive e and decreases slowly for a negative e (see Fig. 1), which can be derived from equation (13). Larger ω changes the effective potential more. The dependence of effective potential for Dirac fields on ω is more complex than for scalar fields. In contrary to scalar fields, small ω changes the effective potential more, especially for $eQ > 0$ (see Fig. 2). Integrating equations (5, 6), we obtain the expression of $\hat{r}_*$ and r_* for $r > r_+$

$$\hat{r}_*=r_*+\frac{\arctan\frac{m_pr}{\lambda}}{2\omega}-\frac{eQ\left[r_+\ln(r-r_+)-r_-\ln(r-r_-)\right]}{\omega(r_+-r_-)} \tag{14}$$

$$r_*=r+\frac{r_+^2\ln(r-r_+)}{r_+-r_-}-\frac{r_-^2\ln(r-r_-)}{r_+-r_-}. \tag{15}$$

From equations (14) and (15), the $\hat{r}_*(r)$-relation is single-valued for $r > r_+$ so long as $1 - eQ/r_+ > 0$ which means $eQ < 0$ or $eQ > 0$ and $\omega > eQ/r_+ = \omega_s$.

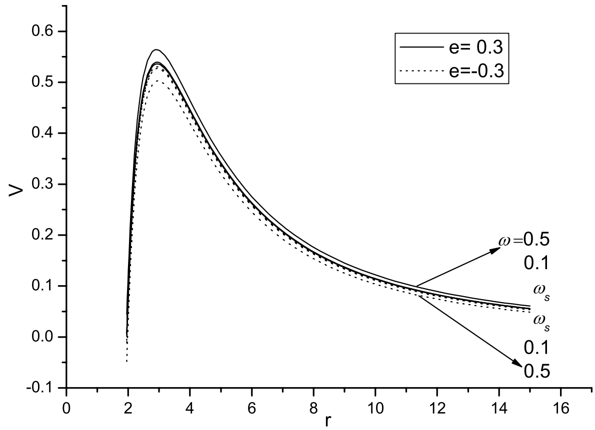

Fig. 1. Variation of effective potential for scalar fields with $l = 3$, $m_p = 0.03$, $Q = 0.3$, $e = \pm 0.3$ and $\omega = 0.5, 0.1, \omega_s$ (defined as Dirac fields).

The super-radiance frequency ω_s is solely determined by the electromagnetic interaction between the Dirac fields and black hole. In this case, the effective potential of Dirac fields varies smoothly. When this inequality obtains, the $\hat{r}_*(r)$-relation is double-valued: $\hat{r}_* \to +\infty$ both when $r \to \infty$ and when $r \to r_+ + 0$ and the effective potential of Dirac fields becomes singular at a certain location $r = \alpha (> r_+)$ when $eQ > 0$ and $\omega < \omega_s$. Wagh (1985) has proved the absence of

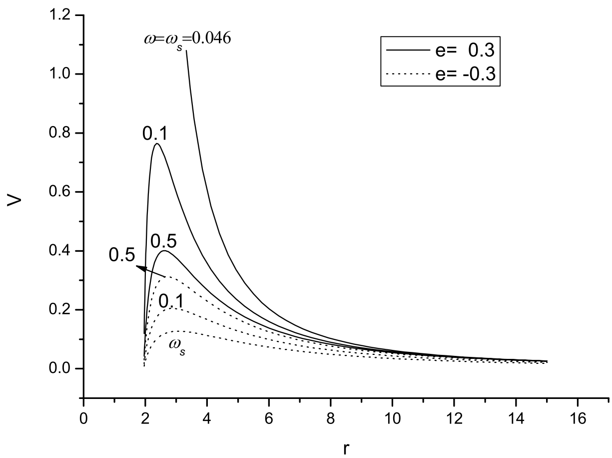

Fig. 2. Variation of effective potential for Dirac fields with $l = 3$, $m_p = 0.03$, $Q = 0.3$, $e = \pm 0.3$ and $\omega = 0.5, 0.1, \omega_s$.

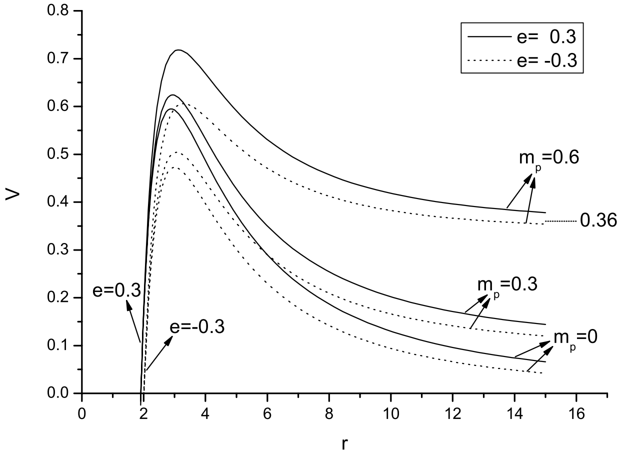

Fig. 3. Variation of effective potential for scalar fields with $l = 3$, $\omega = 1$, $Q = 0.3$, $e = \pm 0.3$ and $m_p = 0, 0.3, 0.6$.

super-radiance (Chandrasekhar, 1983) of Dirac fields in Kerr-Newman geometry which goes over to the RN geometry although $eQ > 0$ and $\omega < \omega_s$.

Figures 3 and 4 show the dependence of the effective potentials on the mass of scalar and Dirac fields. When $r \to r_+$, the effective potential of scalar fields $V \to 2\omega eQ/r_+ - e^2Q^2/r_+^2$, which is different from Dirac case, $V \to 0$. The effective potentials of scalar and Dirac fields approach the same constant value,

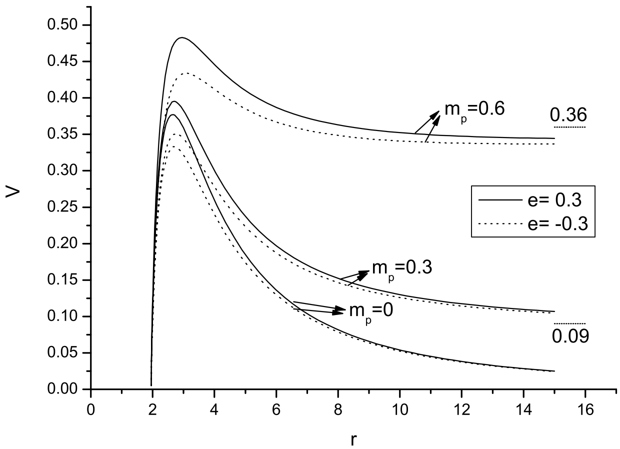

Fig. 4. Variation of effective potential for Dirac fields with $l = 3$, $\omega = 1$, $Q = 0.3$, $e = \pm 0.3$ and $m_p = 0, 0.3, 0.6$.

m_p^2, with different tendencies at spatial infinity which can be described as follow:

$$V_{r\to\infty} = \begin{cases} m_p^2 - 2\frac{Mm_p^2}{r} + 2\frac{eQm_p^2}{r\omega} + O\left(\frac{1}{r^2}\right) & \text{(Dirac)} \\ m_p^2 - 2\frac{Mm_p^2}{r} + 2\frac{eQ\omega}{r} + O\left(\frac{1}{r^2}\right) & \text{(scalar)} \end{cases} \tag{16}$$

3. EVALUATION OF QNMS AND DISCUSSION

The WKB method was proposed by Schutz and Will (1985) for the lowest order and extended by Iyer and Will (1987) to the third modes and Konoplya (2003) to the sixth order. The WKB method can be generalized to RN and Kerr black holes and the third order approximation yield very accurate results for the low-lying modes. In this paper, we compute the QMMs using the third order WKB formula:

$$\omega^2 = [V_0 + (-2V_0'')^{1/2}\Delta] - i\left(n + \frac{1}{2}\right)(-2V_0'')^{1/2}(1 + \Omega), \tag{17}$$

where

$$\alpha = n + \frac{1}{2}, \quad n = \begin{cases} 0, 1, 2, \ldots, \mathrm{Re}(\omega) > 0, \\ -1, -2, -3, \ldots, \mathrm{Re}(\omega) < 0, \end{cases} \tag{18}$$

$$V_0^{(n)} = \left.\frac{\mathrm{d}^n V}{\mathrm{d}\hat{r}_*^n}\right|_{\hat{r}_* = \hat{r}_*(r_{\max})}, \tag{19}$$

n is the mode number and $n < l$ for low-lying modes. Δ and Ω are complex expressions of $V_0^{(n)}$ which can be found in Iyer and Will (1987) and many other literatures.

Since the effective potentials of Dirac fields depend on ω and have more complex form than scalar fields, the procedure of finding scalar QNMs used by Konoplya (2002) can not work well. The procedure need to find the value of $r = r_{\max}$ at which the effective potential attains the maximum as a numerical function of ω. Seidel and Iyer (1990) used Taylor expansions of both the effective potential and the quantity $r_{\max}$, which both involve the complex frequency ω, to calculate the QNMs of Kerr black holes using the similar method, Cho (2003) evaluated the massive Dirac QNMs of Schwarzschild black holes. But the error of this method increases as the quantity of power series increase. Using Muller's method based on quadratic interpolation among three points, Kokkotas (1991) solved the system of two complex transcendental equations and evaluated the QNMs of Kerr black holes. The results of Kokkotas is more accurate than that of Seidel and Iyer for large a, the angular momentum per unit mass of rotating black holes. The Minpack routine HYBRD is to find a zero of a system of N non-linear function in N variables by a modification of Powell hybrid method. In this paper, we adopt the double precision version of HYBRD program to evaluate the QNMs.

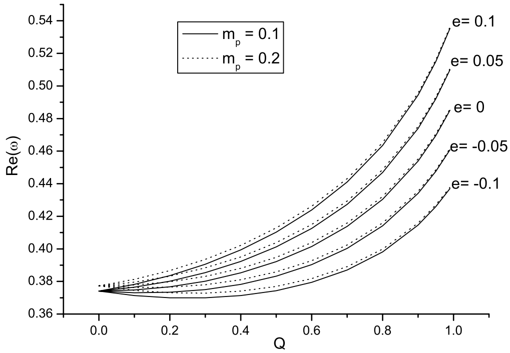

Fig. 5. Re(ω) as function of Q for Dirac fields with $l = 2$, $n = 0$, $e = 0, \pm0.05, \pm0.1$ and $m_p = 0.1, 0.2$.

Figures 5, 6, 7 and 8 show the real and imaginary parts of ω as a function of Q for $l = 2$, $n = 0$, $m_p = 0.1, 0.2$ and $e = 0, \pm0.05, \pm0.1$. In these paragraphs, we discuss under the condition of small magnitude of fields charges. In general speaking, the modifications of the masses on real and imaginary parts decrease as Q increases. For $Q = 0$, the RN black hole becomes a Schwarzschild black hole, the QNMs of scalar and Dirac fields are both determined by m_p, the masses of the fields. Without the Coulomb interactions between the charged fields and black hole, the charges of fields do not change the QNMs. For a given l, Re(ω) grows with increasing Q while e larger than a critical e_0, which determined by l. If $e < e_0$, Re(ω) decreases first then increase. As the Q increases, the influence of e gets larger. For fixed Q and m_p, Re(ω) grows with increasing e. The detailed discussion on the influence of e and m_p are in the following paragraphs. When $Q \to 1$, a

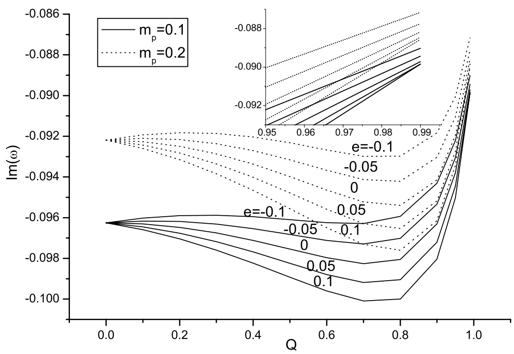

Fig. 6. Im(ω) as a function of Q for Dirac fields with $l = 2$, $n = 0$, $e = 0, \pm0.05, \pm0.1$ and $m_p = 0.1, 0.2$.

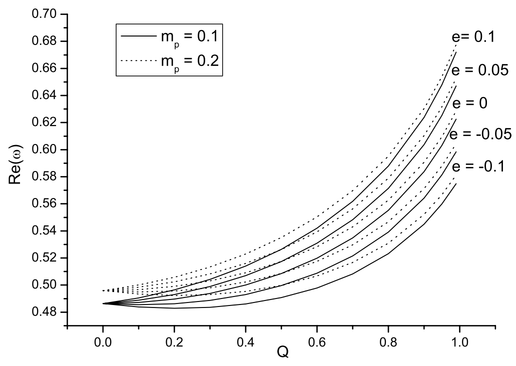

Fig. 7. Re(ω) as a function of Q for scalar fields with $l = 2, n = 0, e = 0, \pm 0.05, \pm 0.1$ and $m_p = 0.1, 0.2$.

near extremal RN black hole, the real parts of QNMs determined by the small magnitude charges of fields (Figs. 5 and 7). As the increase of Q, the influences on Im(ω) of e reaches a maximum then decreases. There also exits a critical e_0', the magnitude of Im(ω) increases first then decreases and increases later when $e < e_0'$. For fixed Q and m_p, the magnitude of Im(ω) increases monotonically with e. In the enlarged views of Figs. 6 and 8, for the near extremal RN black hole the approach values of Im(ω) depend on the masses of fields.

In Figs. 9, 10, 11 and 12, we show the QNMS as a function of m_p. Ohashi and Sakagami (2004) found the quasi-resonances modes (QRMs) for the massive scalar field using Leaver method, which means Im($\omega \to 0$) and the perturbation with arbitrary life. Konoplya and Zhidenko (2005) proved that the purely real frequencies are not forbidden for massive scalar field and study the overtones

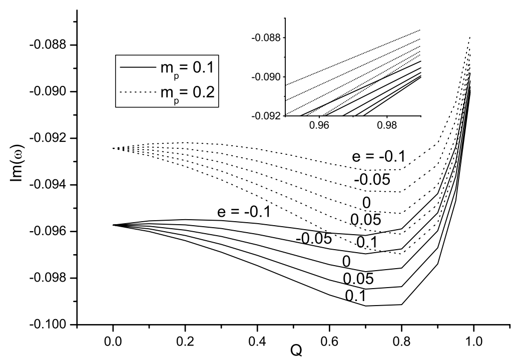

Fig. 8. Im(ω) as a function of Q for scalar fields with $l = 2, n = 0, e = 0, \pm 0.05, \pm 0.1$ and $m_p = 0.1, 0.2$.

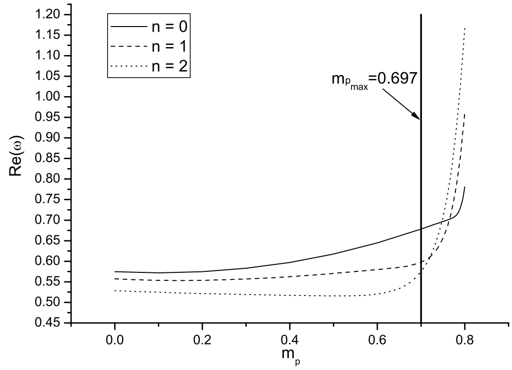

Fig. 9. Re(ω) as a function of m_p for Dirac fields with $l = 3, n = 0, 1, 2, e = 0$ and $Q = 0.1$.

QNMs of scalar field in Schwarzschild black holes using Leaver method. We study the variations of scalar and Dirac low-lying QNMs for the masses of fields using WKB method. The larger m_p increases the tails of effective potentials and broadens the effective potential peak, which will add inaccuracy of WKB method. For the low-lying QNMs, tunnelling to occur ω^2 must be smaller than the peak value of the potential and the energies of the fields are always larger than the masses m_p, so the QNMs exist only when $m^2 < \omega^2 < V_{\max}$. We can estimate the maximum value $m_{p\ \max}$ from Simone and Will (1992); Cho (2003):

$$V(r_{\max}, \omega = m_{p\ \max}) = (m_{p\ \max})^2. \tag{20}$$

As the masses of scalar and Dirac fields increase, the real parts of QNMs increase and the magnitude of imaginary parts decreases. Near the $m_{p\ \max}$ derived from

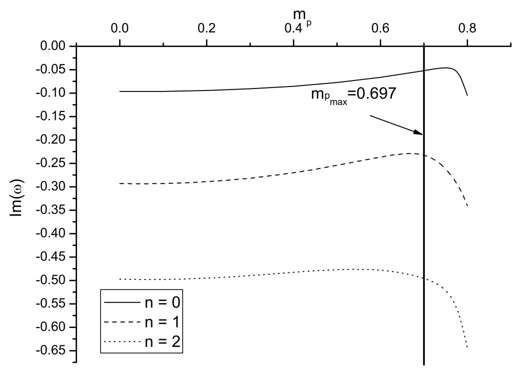

Fig. 10. Im(ω) as a function of m_p for Dirac fields with $l = 3, n = 0, 1, 2, e = 0$ and $Q = 0.1$.

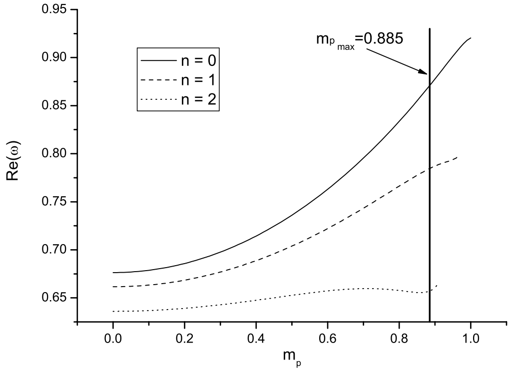

Fig. 11. Re(ω) as a function of m_p for scalar fields with $l = 3, n = 0, 1, 2, e = 0$ and $Q = 0.1$.

equation (20), the real parts of higher tones Dirac fields QNMs increase rapidly first then the lower tones (Fig. 9), and for scalar fields case, there exist abnormal decrease (for $n = 2$) or increase (for $n = 0, 1$, Fig. 11). The magnitude of imaginary parts increases correspondingly at the same value of m_p where the real parts of QNMs become abnormal. Because the larger value of m_p add inaccuracy of WKB method, the abnormal changes of QNMs vary with increasing masses of fields mean the WKB method out of work. Larger n modes change abnormally firstly accord with the fact that the WKB method is more accurate for the fundamental modes. We will discuss this in another article.

Figures 13, 14, 15, and 16 show the QNMs variation with e. In general, the real parts of scalar and Dirac QNMs both increase linearly as e increases (Figs. 13 and 15). For the large value of Q, the decrease of e can lead to tiny values of the Re(ω), which means damp without oscillations (Konoplya, 2006). The imaginary

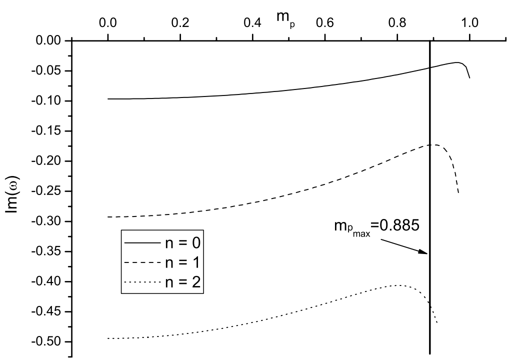

Fig. 12. Im(ω) as a function of m_p for scalar fields with $l = 3, n = 0, 1, 2, e = 0$ and $Q = 0.1$.

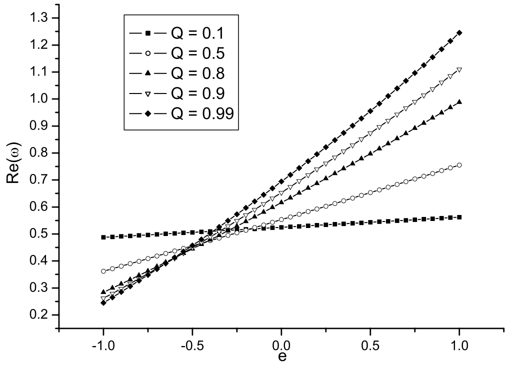

Fig. 13. Re(ω) as a function of e for Dirac fields with $l = 3, n = 2, Q = 0.1, 0.5, 0.8, 0.9, 0.99$ and $m_p = 0.1$.

parts decrease as e increases except the near extremal RN black hole. For the near extremal RN black hole, the imaginary parts increase as e increases when e larger than a special positive value (Figs. 14 and 16).

4. SUMMARY

In this paper, we have discussed the charged massive scalar and Dirac fields QNMs by using WKB method. We first derived the one-dimensional Schrödinger-like equations of scalar and Dirac fields in RN black holes. The expression of Dirac fields is more complex than scalar fields. The approach values of potentials when $r \to r_+$ and $r \to +\infty$ are different. Variations of potential on ω and m_p

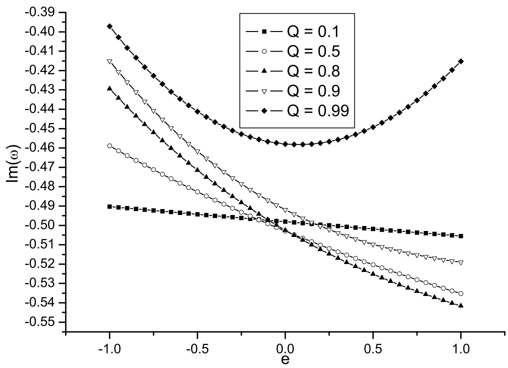

Fig. 14. Im(ω) as a function of e for Dirac fields with $l = 3, n = 2, Q = 0.1, 0.5, 0.8, 0.9, 0.99$ and $m_p = 0.1$.

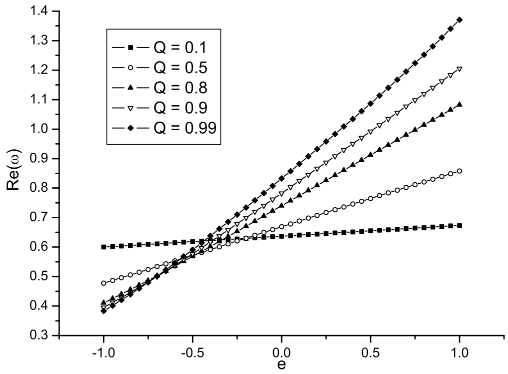

Fig. 15. Re(ω) as a function of e for scalar fields with $l = 3, n = 2, Q = 0.1, 0.5, 0.8, 0.9, 0.99$ and $m_p = 0.1$.

are plotted in Figs. 1, 2, 3, and 4. The Dirac fields have super-radiance frequency ω_s. When $eQ > 0$ and $\omega < \omega_s$, the $\hat{r}_*(r)$-relation is double-valued, which make potentials of Dirac fields singular. We use double precision version of HYBRD program to evaluate the QNMs instead of series expansion method.

Though the different coordinate transformation and effective potentials for scalar and Dirac fields, the behavior of their QNMs are very similar. The WKB method can not applied for the large masses fields in the asymptotic flat spacetime. Many results are similar to the references (Konoplya and Zhidenko, 2005; Ohashi and Sakagami, 2004; Wu and Zhao, 2004; Konoplya, 2002).

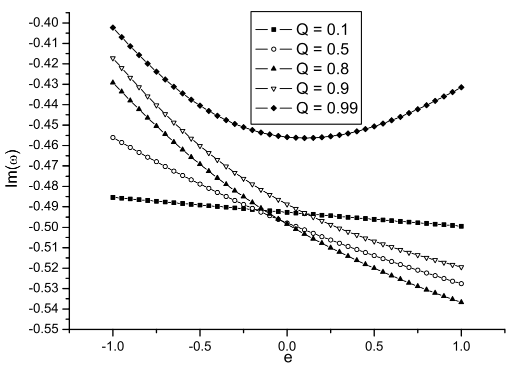

Fig. 16. Im(ω) as a function of e for scalar fields with $l = 3, n = 2, Q = 0.1, 0.5, 0.8, 0.9, 0.99$ and $m_p = 0.1$.

ACKNOWLEDGMENTS

The work has been supported by the National Natural Science Foundation of China (Grant No. 10573027) and Natural Science Foundation of Shanghai (Grant No. 05ZR14138). J. F. Chang thanks Dr. Xian-Hui Ge for his zealous help during the work.

REFERENCES

Anderson, A. and Price, R. H. (1991). *Physical Review D* **43**, 3147.
Berti, E. and Kokkotas, K. D. (2005). *Physical Review D* **71**, 124008.
Cardoso, V. and Lemos, J. P. (2003). *Physical Review D* **67**, 084020.
Cardoso, V., Natario, J., and Schiappa, R. (2004). *Journal of Mathematical Physics* **45**, 4698.
Chandrasekhar, S. (1983). *The Mathematical Theory of Black Holes*, Oxford University Press, Oxford, UK.
Chang, J. F. and Shen, Y. G. (2005). *Nuclear Physics B* **712**, 347.
Cho, H. T. (2003). *Physical Review D* **68**, 024003.
Daghigh, R. G. and Kunstatter, G. (2005). *Classical and Quantum Gravity* **22**, 4113.
Goldberg, J. N., *et al*. (1967). *Journal of Mathematical Physics* **8**, 2155.
Iyer, S. and Will, C. M. (1987). *Physical Review D* **35**, 3621.
Jing, J. L. (2005). *Physical Review D* **71**, 124006.
Jing, J. L. and Pan, Q. Y. (2005a). *Nuclear Physics B* **728**, 109.
Jing, J. L. and Pan, Q. Y. (2005b). *Physical Review D* **71**, 124011.
Kokkotas, K. D. (1991). *Classical and Quantum Gravity* **8**, 2217.
Kokkotas, K. D. and Schmidt, B. G. (1999). *Living Reviews in Relativity* **2**, 2.
Konoplya, R. A. (2002). *Physics Letters B* **550**, 117.
Konoplya, R. A. (2003). *Physical Review D* **68**, 024018.
Konoplya, R. A. (2006). *Physical Review D* **73**, 024009.
Konoplya, R. A. and Abdalla, E. (2005). *Physical Review D* **71**, 084015.
Konoplya, R. A. and Zhidenko, A. V. (2005). *Physics Letters B* **609**, 377.
Nollert, H. P. (1999). *Classical and Quantum Gravity* **16**, R159.
Ohashi, A. and Sakagami, M. (2004). *Classical and Quantum Gravity* **21**, 3973.
Page, D. N. (1976). *Physical Review D* **14**, 1509.
Schutz, B. F. and Will, C. M. (1985). *Astrophysical Journal Letters* **291**, L33.
Seidel, E. and Iyer, S. (1990). *Physical Review D* **41**, 374.
Simone, L. E. and Will, C. M. (1992). *Classical and Quantum Gravity* **9**, 963.
Su, F. W. and Shen, Y. G. (2004). *Physical Review D* **70**, 084046.
Su, F. W. and Shen, Y. G. (2005a). *Physics Letters B* **614**, 195.
Su, F. W. and Shen, Y. G. (2005b). *Physics Letters B* **619**, 340.
Wagh, S. M. and Dadhic, N. (1985). *Physical Review D* **32**, 1863.
Wu, Y. J. and Zhao, Z. (2004). *Physical Review D* **69**, 084015.
Xue, L. H., Wang, B., and Shu, R. K. (2002). *Physical Review D* **66**, 024032.
Zhidenko, A. V. (2004). *Classical and Quantum Gravity* **21**, 273.
Zhidenko, A. V. (2006). *Classical and Quantum Gravity* **23**, 3155.